Constrained Optimization, Duality

CSci 8980: ML at Large Scale and High Dimensions

Instructor: Arindam Banerjee

January 29, 2014

Constrained Optimization

- The equality & inequality constrained optimization problem

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & h_i(\mathbf{x}) = 0 \qquad i = 1, \dots, m \\ & g_j(\mathbf{x}) \le 0 \qquad j = 1, \dots, n \end{aligned}$$

Constrained Optimization

- The equality & inequality constrained optimization problem

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & h_i(\mathbf{x}) = 0 \qquad i = 1, \ldots, m \\ & g_j(\mathbf{x}) \le 0 \qquad j = 1, \ldots, n \end{aligned}$$

  - Domain $\mathcal{D} = \text{dom}(f) \cap \bigcap_{i=1}^{m} \text{dom}(h_i) \cap \bigcap_{j=1}^{n} \text{dom}(g_j)$

Constrained Optimization

- The equality & inequality constrained optimization problem

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & h_i(\mathbf{x}) = 0 \qquad i = 1, \ldots, m \\ & g_j(\mathbf{x}) \leq 0 \qquad j = 1, \ldots, n \end{aligned}$$

  - Domain $\mathcal{D} = \text{dom}(f) \cap \bigcap_{i=1}^{m} \text{dom}(h_i) \cap \bigcap_{j=1}^{n} \text{dom}(g_j)$
- The Lagrangian

$$\begin{aligned} L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\nu}) = & \; f(\mathbf{x}) + \boldsymbol{\lambda}^T h(\mathbf{x}) + \boldsymbol{\nu}^T g(\mathbf{x}) \\ = & \; f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i h_i(\mathbf{x}) + \sum_{j=1}^{n} \nu_j g_j(\mathbf{x}) \end{aligned}$$

Constrained Optimization

- The equality & inequality constrained optimization problem

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & h_i(\mathbf{x}) = 0 \qquad i = 1, \dots, m \\ & g_j(\mathbf{x}) \leq 0 \qquad j = 1, \dots, n \end{aligned}$$

  - Domain $\mathcal{D} = \text{dom}(f) \cap \bigcap_{i=1}^{m} \text{dom}(h_i) \cap \bigcap_{j=1}^{n} \text{dom}(g_j)$
- The Lagrangian

$$\begin{aligned} L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\nu}) = & \; f(\mathbf{x}) + \boldsymbol{\lambda}^T h(\mathbf{x}) + \boldsymbol{\nu}^T g(\mathbf{x}) \\ = & \; f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i h_i(\mathbf{x}) + \sum_{j=1}^{n} \nu_j g_j(\mathbf{x}) \end{aligned}$$

  - Domain $\text{dom}(L) = \mathcal{D} \times \mathbb{R}^m \times \mathbb{R}^n$

Constrained Optimization

- The equality & inequality constrained optimization problem

$$\begin{aligned} \text{minimize} \quad & f(\mathbf{x}) \\ \text{subject to} \quad & h_i(\mathbf{x}) = 0 \qquad i = 1, \ldots, m \\ & g_j(\mathbf{x}) \leq 0 \qquad j = 1, \ldots, n \end{aligned}$$

  - Domain $\mathcal{D} = \text{dom}(f) \cap \bigcap_{i=1}^m \text{dom}(h_i) \cap \bigcap_{j=1}^n \text{dom}(g_j)$
- The Lagrangian

$$\begin{aligned} L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\nu}) = & \ f(\mathbf{x}) + \boldsymbol{\lambda}^T h(\mathbf{x}) + \boldsymbol{\nu}^T g(\mathbf{x}) \\ = & \ f(\mathbf{x}) + \sum_{i=1}^m \lambda_i h_i(\mathbf{x}) + \sum_{j=1}^n \nu_j g_j(\mathbf{x}) \end{aligned}$$

  - Domain $\text{dom}(L) = \mathcal{D} \times \mathbb{R}^m \times \mathbb{R}^n$
- $\{\lambda_i\}_{i=1}^m, \{\nu_j\}_{j=1}^n$ are the Lagrange multipliers

Lagrange Dual

- The Lagrange dual function

$$\begin{aligned} L^*(\boldsymbol{\lambda}, \boldsymbol{\nu}) &= \inf_{\mathbf{x} \in \mathcal{D}} \; L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\nu}) \\ &= \inf_{\mathbf{x} \in \mathcal{D}} \; \left(f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i h_i(\mathbf{x}) + \sum_{j=1}^{n} \nu_j g_j(\mathbf{x}) \right) \end{aligned}$$

Lagrange Dual

- The Lagrange dual function

$$\begin{aligned} L^*(\boldsymbol{\lambda}, \boldsymbol{\nu}) &= \inf_{\mathbf{x} \in \mathcal{D}} L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\nu}) \\ &= \inf_{\mathbf{x} \in \mathcal{D}} \left(f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i h_i(\mathbf{x}) + \sum_{j=1}^{n} \nu_j g_j(\mathbf{x}) \right) \end{aligned}$$

- Let p^* be the constrained optimum of $f(\mathbf{x})$

Lagrange Dual

- The Lagrange dual function

$$\begin{aligned} L^*(\boldsymbol{\lambda}, \boldsymbol{\nu}) &= \inf_{\mathbf{x} \in \mathcal{D}} L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\nu}) \\ &= \inf_{\mathbf{x} \in \mathcal{D}} \left(f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i h_i(\mathbf{x}) + \sum_{j=1}^{n} \nu_j g_j(\mathbf{x}) \right) \end{aligned}$$

- Let p^* be the constrained optimum of $f(\mathbf{x})$
- The Lagrange dual L^* is

Lagrange Dual

- The Lagrange dual function

$$
\begin{aligned}
L^*(\boldsymbol{\lambda},\boldsymbol{\nu}) &= \inf_{\mathbf{x}\in\mathcal{D}} L(\mathbf{x},\boldsymbol{\lambda},\boldsymbol{\nu}) \\
&= \inf_{\mathbf{x}\in\mathcal{D}} \left(f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i h_i(\mathbf{x}) + \sum_{j=1}^{n} \nu_j g_j(\mathbf{x}) \right)
\end{aligned}
$$

- Let p^* be the constrained optimum of $f(\mathbf{x})$
- The Lagrange dual L^* is
  - A concave function, even when original problem is not convex

Lagrange Dual

- The Lagrange dual function

$$L^*(\boldsymbol{\lambda}, \boldsymbol{\nu}) = \inf_{\mathbf{x} \in \mathcal{D}} L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\nu})$$
$$= \inf_{\mathbf{x} \in \mathcal{D}} \left(f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i h_i(\mathbf{x}) + \sum_{j=1}^{n} \nu_j g_j(\mathbf{x}) \right)$$

- Let p^* be the constrained optimum of $f(\mathbf{x})$
- The Lagrange dual L^* is
  - A concave function, even when original problem is not convex
  - A lower bound: for $\boldsymbol{\nu} \geq 0$, $L^*(\boldsymbol{\lambda}, \boldsymbol{\nu}) \leq p^*$

Lagrange Dual

- The Lagrange dual function

$$\begin{aligned} L^*(\boldsymbol{\lambda}, \boldsymbol{\nu}) &= \inf_{\mathbf{x} \in \mathcal{D}} L(\mathbf{x}, \boldsymbol{\lambda}, \boldsymbol{\nu}) \\ &= \inf_{\mathbf{x} \in \mathcal{D}} \left(f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i h_i(\mathbf{x}) + \sum_{j=1}^{n} \nu_j g_j(\mathbf{x}) \right) \end{aligned}$$

- Let p^* be the constrained optimum of $f(\mathbf{x})$
- The Lagrange dual L^* is
  - A concave function, even when original problem is not convex
  - A lower bound: for $\boldsymbol{\nu} \geq 0$, $L^*(\boldsymbol{\lambda}, \boldsymbol{\nu}) \leq p^*$
- How close is the maximum of $L^*(\boldsymbol{\lambda}, \boldsymbol{\nu})$ to p^*?

An Example

$$\text{minimize} \quad \mathbf{x}^T\mathbf{x}$$
$$\text{subject to} \quad A\mathbf{x} = b$$

- Lagrangian $L(\mathbf{x}, \boldsymbol{\lambda}) = \mathbf{x}^T\mathbf{x} + \boldsymbol{\lambda}^T(A\mathbf{x} - b)$
- Recall that $L^*(\boldsymbol{\lambda}) = \inf_{\mathbf{x}} \ L(\mathbf{x}, \boldsymbol{\lambda})$
- Setting gradient to 0, $\mathbf{x} = -\frac{1}{2}A^T\boldsymbol{\lambda}$
- Hence, the dual

$$L^*(\boldsymbol{\lambda}) = L\left(-\frac{1}{2}A^T\boldsymbol{\lambda}, \boldsymbol{\lambda}\right) = -\frac{1}{4}\boldsymbol{\lambda}^T A A^T \boldsymbol{\lambda} - \boldsymbol{\lambda}^T b$$

- $L^*(\boldsymbol{\lambda})$ is a lower bounding concave function

The Lagrange Dual Problem

$$\text{maximize } L^*(\boldsymbol{\lambda}, \boldsymbol{\nu})$$
$$\text{subject to } \boldsymbol{\nu} \geq 0$$

- Best lower bound to p^*, the optimal of the primal

The Lagrange Dual Problem

$$\text{maximize } L^*(\boldsymbol{\lambda}, \boldsymbol{\nu})$$
$$\text{subject to } \boldsymbol{\nu} \geq 0$$

- Best lower bound to p^*, the optimal of the primal
- Concave optimization problem with maximum d^*

The Lagrange Dual Problem

$$\text{maximize } L^*(\boldsymbol{\lambda}, \boldsymbol{\nu})$$
$$\text{subject to } \boldsymbol{\nu} \geq 0$$

- Best lower bound to p^*, the optimal of the primal
- Concave optimization problem with maximum d^*
- Constraints are $\boldsymbol{\nu} \geq 0$ and $(\boldsymbol{\lambda}, \boldsymbol{\nu}) \in \text{dom}(L^*)$

The Lagrange Dual Problem

$$\text{maximize } L^*(\boldsymbol{\lambda}, \boldsymbol{\nu})$$
$$\text{subject to } \boldsymbol{\nu} \geq 0$$

- Best lower bound to p^*, the optimal of the primal
- Concave optimization problem with maximum d^*
- Constraints are $\boldsymbol{\nu} \geq 0$ and $(\boldsymbol{\lambda}, \boldsymbol{\nu}) \in \text{dom}(L^*)$
- For example, in linear programming

$$\begin{aligned} \text{minimize } & \mathbf{c}^T\mathbf{x} \\ \text{subject to } & A\mathbf{x} = \mathbf{b} \\ & \mathbf{x} \geq 0 \end{aligned} \qquad \begin{aligned} \text{maximize } & -\mathbf{b}^T\boldsymbol{\lambda} \\ \text{subject to } & A^T\boldsymbol{\lambda} + \mathbf{c} \geq 0 \end{aligned}$$

Weak and Strong Duality

- Weak Duality: $d^* \leq p^*$

Weak and Strong Duality

- Weak Duality: $d^* \leq p^*$
    - Always holds

Weak and Strong Duality

- Weak Duality: $d^* \leq p^*$
  - Always holds
  - Non-trivial lower bounds for hard problems

Weak and Strong Duality

- Weak Duality: $d^* \leq p^*$
  - Always holds
  - Non-trivial lower bounds for hard problems
  - Used in approximation algorithms

Weak and Strong Duality

- Weak Duality: $d^* \leq p^*$
  - Always holds
  - Non-trivial lower bounds for hard problems
  - Used in approximation algorithms
- Strong Duality: $d^* = p^*$

Weak and Strong Duality

- Weak Duality: $d^* \leq p^*$
  - Always holds
  - Non-trivial lower bounds for hard problems
  - Used in approximation algorithms
- Strong Duality: $d^* = p^*$
  - Does not hold in general

Weak and Strong Duality

- Weak Duality: $d^* \leq p^*$
  - Always holds
  - Non-trivial lower bounds for hard problems
  - Used in approximation algorithms
- Strong Duality: $d^* = p^*$
  - Does not hold in general
  - If it holds, it is sufficient to solve the dual

Weak and Strong Duality

- Weak Duality: $d^* \leq p^*$
  - Always holds
  - Non-trivial lower bounds for hard problems
  - Used in approximation algorithms
- Strong Duality: $d^* = p^*$
  - Does not hold in general
  - If it holds, it is sufficient to solve the dual
  - How to check it if holds?

Weak and Strong Duality

- Weak Duality: $d^* \leq p^*$
  - Always holds
  - Non-trivial lower bounds for hard problems
  - Used in approximation algorithms
- Strong Duality: $d^* = p^*$
  - Does not hold in general
  - If it holds, it is sufficient to solve the dual
  - How to check it if holds?
- Constraint Qualification

Weak and Strong Duality

- Weak Duality: $d^* \leq p^*$
  - Always holds
  - Non-trivial lower bounds for hard problems
  - Used in approximation algorithms
- Strong Duality: $d^* = p^*$
  - Does not hold in general
  - If it holds, it is sufficient to solve the dual
  - How to check it if holds?
- Constraint Qualification
  - Normally true on convex problems

Weak and Strong Duality

- Weak Duality: $d^* \leq p^*$
  - Always holds
  - Non-trivial lower bounds for hard problems
  - Used in approximation algorithms
- Strong Duality: $d^* = p^*$
  - Does not hold in general
  - If it holds, it is sufficient to solve the dual
  - How to check it if holds?
- Constraint Qualification
  - Normally true on convex problems
  - True if the convex problem is strictly feasible, e.g.,

$$\exists x \in \text{relint}(\mathcal{D}) \quad \text{s.t.} \quad Ax = b, \quad g_j(x) < 0, \quad \text{for some } j$$

Weak and Strong Duality

- Weak Duality: $d^* \leq p^*$
  - Always holds
  - Non-trivial lower bounds for hard problems
  - Used in approximation algorithms
- Strong Duality: $d^* = p^*$
  - Does not hold in general
  - If it holds, it is sufficient to solve the dual
  - How to check it if holds?
- Constraint Qualification
  - Normally true on convex problems
  - True if the convex problem is strictly feasible, e.g.,

$$\exists x \in \text{relint}(\mathcal{D}) \quad \text{s.t.} \quad Ax = b, \quad g_j(x) < 0, \quad \text{for some } j$$

  - Slater's Condition for strong duality

Example: Quadratic Programs

$$\begin{aligned} \text{minimize} \quad & \mathbf{x}^T \mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} \leq \mathbf{b} \end{aligned}$$

- Lagrange dual

$$L^*(\boldsymbol{\nu}) = \inf_{\mathbf{x}} \left(\mathbf{x}^T \mathbf{x} + \boldsymbol{\nu}^T (A\mathbf{x} - \mathbf{b}) \right) = -\frac{1}{4} \boldsymbol{\nu}^T A A^T \boldsymbol{\nu} - b^T \boldsymbol{\nu}$$

- Dual problem

$$\begin{aligned} \text{maximize} \quad & -\frac{1}{4} \boldsymbol{\nu}^T A A^T \boldsymbol{\nu} - b^T \boldsymbol{\nu} \\ \text{subject to} \quad & \boldsymbol{\nu} \geq 0 \end{aligned}$$

- From Slater's condition, $p^* = d^*$
- It is sufficient to solve the dual

Complementary Slackness

- If strong duality holds, $\mathbf{x}^*$ for primal, $(\boldsymbol{\lambda}^*, \boldsymbol{\nu}^*)$ for dual

$$
\begin{aligned}
f(\mathbf{x}^*) = L^*(\boldsymbol{\lambda}^*, \boldsymbol{\nu}^*) = & \inf_{\mathbf{x}} \left(f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i^* h_i(\mathbf{x}) + \sum_{j=1}^{n} \nu_j^* g_j(\mathbf{x}) \right) \\
\leq & f(\mathbf{x}^*) + \sum_{i=1}^{m} \lambda_i^* h_i(\mathbf{x}^*) + \sum_{j=1}^{n} \nu_j^* g_j(\mathbf{x}^*) \\
\leq & f(\mathbf{x}^*)
\end{aligned}
$$

Complementary Slackness

- If strong duality holds, $\mathbf{x}^*$ for primal, $(\boldsymbol{\lambda}^*, \boldsymbol{\nu}^*)$ for dual

$$
\begin{aligned}
f(\mathbf{x}^*) = L^*(\boldsymbol{\lambda}^*, \boldsymbol{\nu}^*) = & \inf_{\mathbf{x}} \left(f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i^* h_i(\mathbf{x}) + \sum_{j=1}^{n} \nu_j^* g_j(\mathbf{x}) \right) \\
\leq & f(\mathbf{x}^*) + \sum_{i=1}^{m} \lambda_i^* h_i(\mathbf{x}^*) + \sum_{j=1}^{n} \nu_j^* g_j(\mathbf{x}^*) \\
\leq & f(\mathbf{x}^*)
\end{aligned}
$$

- The two inequalities hold with equality

Complementary Slackness

- If strong duality holds, $\mathbf{x}^*$ for primal, $(\boldsymbol{\lambda}^*, \boldsymbol{\nu}^*)$ for dual

$$
\begin{aligned}
f(\mathbf{x}^*) = L^*(\boldsymbol{\lambda}^*, \boldsymbol{\nu}^*) = & \inf_{\mathbf{x}} \left(f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i^* h_i(\mathbf{x}) + \sum_{j=1}^{n} \nu_j^* g_j(\mathbf{x}) \right) \\
\leq & \; f(\mathbf{x}^*) + \sum_{i=1}^{m} \lambda_i^* h_i(\mathbf{x}^*) + \sum_{j=1}^{n} \nu_j^* g_j(\mathbf{x}^*) \\
\leq & \; f(\mathbf{x}^*)
\end{aligned}
$$

- The two inequalities hold with equality
  - $\mathbf{x}^*$ minimizes the Lagrangian $L(\mathbf{x}, \boldsymbol{\lambda}^*, \boldsymbol{\nu}^*)$

Complementary Slackness

- If strong duality holds, $\mathbf{x}^*$ for primal, $(\boldsymbol{\lambda}^*, \boldsymbol{\nu}^*)$ for dual

$$
\begin{aligned}
f(\mathbf{x}^*) = L^*(\boldsymbol{\lambda}^*, \boldsymbol{\nu}^*) = & \inf_{\mathbf{x}} \left(f(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i^* h_i(\mathbf{x}) + \sum_{j=1}^{n} \nu_j^* g_j(\mathbf{x}) \right) \\
\leq & f(\mathbf{x}^*) + \sum_{i=1}^{m} \lambda_i^* h_i(\mathbf{x}^*) + \sum_{j=1}^{n} \nu_j^* g_j(\mathbf{x}^*) \\
\leq & f(\mathbf{x}^*)
\end{aligned}
$$

- The two inequalities hold with equality
  - $\mathbf{x}^*$ minimizes the Lagrangian $L(\mathbf{x}, \boldsymbol{\lambda}^*, \boldsymbol{\nu}^*)$
  - $\nu_j^* g_j(\mathbf{x}^*) = 0$ for all $j = 1, \ldots, n$ so that

$$
\nu_j^* > 0 \Rightarrow g_j(\mathbf{x}^*) = 0, \qquad \text{and} \qquad g_j(\mathbf{x}^*) < 0 \Rightarrow \nu_j^* = 0
$$

Karush-Kuhn-Tucker (KKT) Conditions

Necessary conditions satisfied by any primal and dual optimal pairs $\tilde{\mathbf{x}}$ and $(\tilde{\boldsymbol{\lambda}}, \tilde{\boldsymbol{\nu}})$

- Primal Feasibility:

$$h_i(\tilde{\mathbf{x}}) = 0, i = 1, \ldots, n, \quad g_j(\tilde{\mathbf{x}}) \leq 0, j = 1, \ldots, m$$

Karush-Kuhn-Tucker (KKT) Conditions

Necessary conditions satisfied by any primal and dual optimal pairs $\tilde{\mathbf{x}}$ and $(\tilde{\boldsymbol{\lambda}}, \tilde{\boldsymbol{\nu}})$

- Primal Feasibility:

$$h_i(\tilde{\mathbf{x}}) = 0, i = 1, \ldots, n, \quad g_j(\tilde{\mathbf{x}}) \leq 0, j = 1, \ldots, m$$

- Dual Feasibility:

$$\tilde{\nu}_j \geq 0, j = 1, \ldots, m$$

Karush-Kuhn-Tucker (KKT) Conditions

Necessary conditions satisfied by any primal and dual optimal pairs $\tilde{\mathbf{x}}$ and $(\tilde{\boldsymbol{\lambda}}, \tilde{\boldsymbol{\nu}})$

- Primal Feasibility:

$$h_i(\tilde{\mathbf{x}}) = 0, i = 1, \ldots, n, \quad g_j(\tilde{\mathbf{x}}) \leq 0, j = 1, \ldots, m$$

- Dual Feasibility:

$$\tilde{\nu}_j \geq 0, j = 1, \ldots, m$$

- Complementary Slackness:

$$\tilde{\nu}_j g_j(\tilde{\mathbf{x}}) = 0, j = 1, \ldots, m$$

Karush-Kuhn-Tucker (KKT) Conditions

Necessary conditions satisfied by any primal and dual optimal pairs $\tilde{\mathbf{x}}$ and $(\tilde{\boldsymbol{\lambda}}, \tilde{\boldsymbol{\nu}})$

- Primal Feasibility:

$$h_i(\tilde{\mathbf{x}}) = 0, i = 1, \ldots, n, \quad g_j(\tilde{\mathbf{x}}) \leq 0, j = 1, \ldots, m$$

- Dual Feasibility:

$$\tilde{\nu}_j \geq 0, j = 1, \ldots, m$$

- Complementary Slackness:

$$\tilde{\nu}_j g_j(\tilde{\mathbf{x}}) = 0, j = 1, \ldots, m$$

- Gradient condition:

$$\nabla f(\tilde{\mathbf{x}}) + \sum_{i=1}^{n} \tilde{\lambda}_i \nabla h_i(\tilde{\mathbf{x}}) + \sum_{j=1}^{m} \tilde{\nu}_j \nabla g_j(\tilde{\mathbf{x}}) = 0$$

Karush-Kuhn-Tucker (KKT) Conditions

Necessary conditions satisfied by any primal and dual optimal pairs $\tilde{\mathbf{x}}$ and $(\tilde{\boldsymbol{\lambda}}, \tilde{\boldsymbol{\nu}})$

- Primal Feasibility:

$$h_i(\tilde{\mathbf{x}}) = 0, i = 1, \ldots, n, \quad g_j(\tilde{\mathbf{x}}) \leq 0, j = 1, \ldots, m$$

- Dual Feasibility:

$$\tilde{\nu}_j \geq 0, j = 1, \ldots, m$$

- Complementary Slackness:

$$\tilde{\nu}_j g_j(\tilde{\mathbf{x}}) = 0, j = 1, \ldots, m$$

- Gradient condition:

$$\nabla f(\tilde{\mathbf{x}}) + \sum_{i=1}^{n} \tilde{\lambda}_i \nabla h_i(\tilde{\mathbf{x}}) + \sum_{j=1}^{m} \tilde{\nu}_j \nabla g_j(\tilde{\mathbf{x}}) = 0$$

- The conditions are <u>sufficient</u> for a convex problem